Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Calculus III for Engineers
MAT 2322A - Fall 2011
Final Exam
Professor: Victor G. LeBlanc
Time limit: 3 hours. Closed books. No calculators.

Name:______________________________ ID Number:____________________

Instructions

- This exam has 15 pages and you have 3 hours to complete it.
- This is a closed book exam. Furthermore, all calculators, cell phones, pagers or any other electronic or communication devices are forbidden.
- Read each question carefully before answering.
- Questions 1 to 10 are multiple choice questions. These questions are worth 2 points each and no partial marks are possible. **Please write your answers in the corresponding boxes in the grid below entitled "Answers to multiple choice Qs".**
- Questions 11 to 16 are long answer questions and are worth 5 marks each, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstaple the test.
- Good luck!

Answers to multiple choice Qs

1	2	3	4	5	6	7	8	9	10

Grid below is used for grading (do not write in this grid)

MCQ	11	12	13	14	15	16	Total
/20	/5	/5	/5	/5	/5	/5	/50

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1. Consider the function $f(x, y) = x^4y + 2xy^2$. If we start at the point $(1, 2)$, along which direction should we move in order to obtain the maximum rate of change of this function?

A. $9\vec{i} + 16\vec{j}$

B. 25

C. $16\vec{i} + 9\vec{j}$

D. $\vec{i}$

E. $\vec{j}$

F. this function does not have a maximum rate of change

2. Which of the following corresponds to the equation for the tangent plane to the surface $z = 2x^2 - 4y^3$ at the point $(2, 1, 4)$?

A. $z = 8\,x - 12\,y + 4$

B. $z = 4x\,(x - 2) - 12y^2(y - 2) + 4$

C. $z = 4$

D. $z = 8\,x - 12\,y$

E. $z = 8\,(x - 2) - 12\,(y - 1) + 4$

F. this function is not differentiable at the given point, so there is no tangent plane.

3. Consider the solid region which is bounded by the planes $z = 0$, $z = 10 - x - 2y$, $x = 0$, $x = 1$, $y = 0$ and $y = 2$. This solid has a mass density given by $\delta(x, y) = xy$. Then the total mass of this solid is

A. $\dfrac{20}{3}$

B. $\dfrac{19}{3}$

C. 1

D. 15

E. 7

F. 0

4. Which of the following corresponds to reversing the order of integration for the iterated integral $\displaystyle\int_{x=-1}^{0} \int_{y=x^2}^{1} f(x, y)\, dy\, dx$?

A. $\displaystyle\int_{x=-1}^{0} \int_{y=x^2}^{1} f(y, x)\, dx\, dy$

B. $\displaystyle\int_{y=x^2}^{1} \int_{x=-1}^{0} f(x, y)\, dx\, dy$

C. $\displaystyle\int_{0}^{1} \int_{x=0}^{\sqrt{y}} f(x, y)\, dx\, dy$

D. $\displaystyle\int_{y=0}^{1} \int_{x=-\sqrt{y}}^{0} f(x, y)\, dx\, dy$

E. $\displaystyle\int_{y=0}^{1} \int_{x=-\sqrt{y}}^{0} f(y, x)\, dx\, dy$

F. $\displaystyle\int_{x=0}^{-1} \int_{y=1}^{x^2} f(x, y)\, dy\, dx$

5. Consider the solid region (ice cream cone) bounded above by the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$. This region has a mass density given by $\delta(x, y, z) = x^2 + y^2 + z^2$. Then the total mass of this solid is

A. $\displaystyle\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/4} \int_{\rho=0}^{1} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

B. $\displaystyle\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/4} \int_{\rho=0}^{1} \rho^4 \sin\varphi \, d\rho \, d\varphi \, d\theta$

C. $\displaystyle\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/4} \int_{\rho=0}^{1} \rho^4 \, d\rho \, d\varphi \, d\theta$

D. $\displaystyle\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/4} \int_{\rho=0}^{1} \rho^2 \, d\rho \, d\varphi \, d\theta$

E. $\displaystyle\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/2} \int_{\rho=0}^{1} \rho^4 \sin\varphi \, d\rho \, d\varphi \, d\theta$

F. $\displaystyle\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi/4} \int_{\rho=0}^{1} \rho^3 \sin\varphi \, d\rho \, d\varphi \, d\theta$

6. Consider the parametrized curve $\vec{r}(t) = 2t\,\vec{i} + \sin 3t\,\vec{j} - \cos 3t\,\vec{k}$, $0 \leq t \leq \pi/3$. The total arclength of this curve is

A. $\dfrac{2\pi}{3}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{\pi\sqrt{3}}{3}$

D. $\dfrac{-2\pi}{3}\,\vec{i}$

E. π

F. $\dfrac{\pi\sqrt{13}}{3}$

7. Which of the following is the result of having converted $\displaystyle\int_{x=-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{y=|x|}^{\sqrt{1-x^2}} (x^2 - y)\, dy\, dx$ into polar coordinates?

A. $\displaystyle\int_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, dr\, d\theta$

B. $\displaystyle\int_{\theta=0}^{\frac{\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

C. $\displaystyle\int_{\theta=0}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

D. $\displaystyle\int_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

E. $\displaystyle\int_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \sin^2 \theta - r \cos \theta)\, r\, dr\, d\theta$

F. $\displaystyle\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

8. Let D be a region in the plane on which Green's theorem holds, and let C denote the boundary of D oriented positively with respect to D. Which of the following line integrals will be equal to the area of D? Recall that the area of D is given by $\displaystyle\int\int_D dx\, dy$.

A. $\displaystyle\oint_C x\, dx + y\, dy$

B. $\displaystyle\oint_C y\, dx + x\, dy$

C. $\displaystyle\oint_C x\, dy$

D. $\displaystyle\oint_C y\, dx$

E. $\displaystyle\oint_C x^2\, dx + y^2\, dy$

F. $\displaystyle\oint_C y^2\, dx - x^2\, dy$

9. Which of the following corresponds to the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ of the vector field $\vec{F}(x, y, z) = (y + z)\,\vec{i} + 2y\,\vec{j} - z\,\vec{k}$ along the straight line segment C which starts at the point $(0, 0, 0)$ and ends at the point $(1, 1, 1)$?

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{9}{2}$

E. $\frac{-3}{2}$

F. $\frac{-5}{2}$

10 Which of the following corresponds to the value of the surface (flux) integral $\int\int_S \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F}(x, y, z) = (y + z)\,\vec{i} + 2y\,\vec{j} - z\,\vec{k}$ over the square S defined by $0 \leq x \leq 2$, $0 \leq y \leq 2$, $z = 3$, and oriented in the direction of the vector $\vec{k}$?

A. -11

B. -12

C. -13

D. -14

E. -15

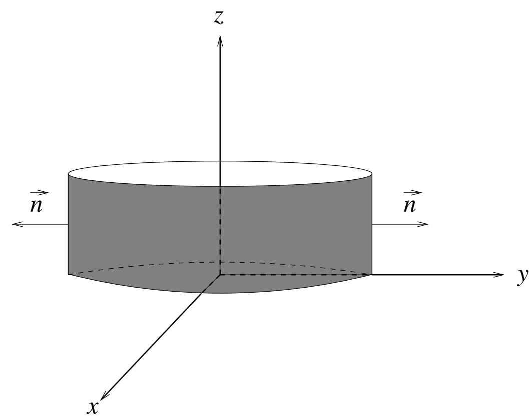
F. -16

11. Consider the vector field $\vec{F}(x,y,z) = (z\,e^x \sin y)\,\vec{i} + (z\,e^x \cos y)\,\vec{j} + (e^x \sin y)\,\vec{k}$. Show that the vector field is conservative, and then find a scalar function $f(x,y,z)$ such that $\vec{F}(x,y,z) = \vec{\nabla} f(x,y,z)$. Finally, compute

$$\int_C \vec{F} \cdot d\vec{r},$$

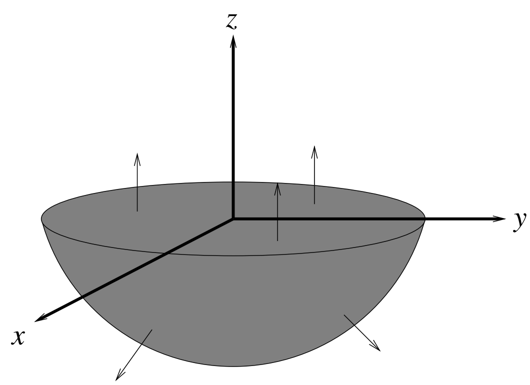
where C is some continuous curve that starts at the point $(0,0,0)$ and ends at the point $\left(1, \frac{\pi}{2}, 2\right)$.

12. Consider the vector field $\vec{F}(x,y,z) = (x-y)\,\vec{i} + (y+x)\,\vec{j} + \vec{k}$. Let S denote the oriented cylindrical strip $x^2 + y^2 = 9$, $0 \le z \le 2$, with normal vector $\vec{n}$ pointing away from the z-axis, as illustrated. Compute the surface (flux) integral $\displaystyle\int\!\!\int_S \vec{F} \cdot d\vec{S}$.

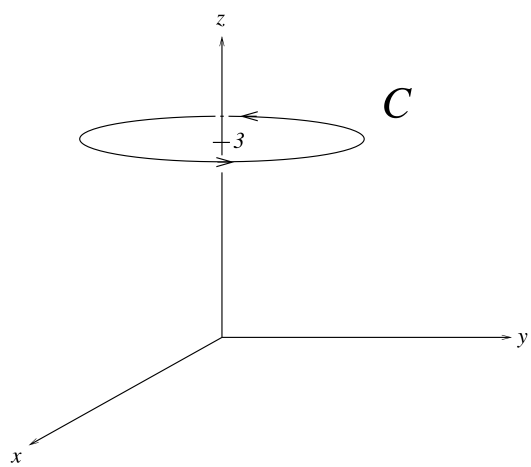


13. Find the global extrema of the function $z = f(x, y) = 3x^2 - 2y^2$ on the disk of radius one, centered at the origin, $0 \leq x^2 + y^2 \leq 1$.

14. For the vector field $\vec{F}(x, y, z) = (x^3 - y^2)\,\vec{i} + (y^3 + x)\,\vec{j} + (z^3 + x)\,\vec{k}$, compute the divergence of $\vec{F}$, i.e. compute $\vec{\nabla} \cdot \vec{F}$. Then, **using Gauss' divergence theorem**, compute the surface (flux) integral $\displaystyle\int\int_S \vec{F} \cdot d\vec{S}$, where S is the oriented surface illustrated below, consisting of two parts: the hemisphere $z = -\sqrt{1 - x^2 - y^2}$, and the disk $0 \leq x^2 + y^2 \leq 1$ in the plane $z = 0$.



15. For the vector field $\vec{F}(x, y, z) = (-y + e^{\sin x})\,\vec{i} + (x - \ln(\cos^2(e^y)))\,\vec{j} + (z^3)\,\vec{k}$, compute the curl of $\vec{F}$, i.e. compute $\vec{\nabla} \times \vec{F}$. Then, **using Stokes theorem**, compute the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where C is the oriented curve $x^2 + y^2 = 1$, $z = 3$, illustrated below.



16. Find and classify the critical points of the function $f(x, y) = x^3 + y^3 + 3xy + 3$.

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